Analysis and Experiment Concerning the Cutoff Frequencies of Rectangular Striplines

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***Abstract*—The cutoff frequency of the first higher order mode in a rectangular stripline is determined by two disparate techniques and compared to published results. Results from a novel experiment verify the cutoff frequency predictions. Furthermore, a nomenclature scheme for uniquely identifying rectangular stripline modes is offered.**

I. Introduction

TEM transmission cells [1], sometimes called Crawford cells [2], are widely used in experiments which require a known, nearly uniform incident electromagnetic field, such as studies on the biological effects of nonionizing radiations. Hill [3] has reported that, at specific frequencies where cavity resonances occur, the field distribution inside TEM cells is altered. From this fact, it has been concluded that the useable bandwidth of TEM cells is restricted to frequency bands separated by cavity resonances [3], [4].

Fig. 1 shows the cross section of a TEM cell, which is a rectangular stripline with a thin inner conductor. Weil and Gruner [4] have published extensive graphs plotting the cutoff frequencies for rectangular striplines and related these cutoff frequencies to the cavity resonances of TEM cells. The basic technique for determining the cutoff frequencies presented in their graphs is the method of Fourier expansion of the fields in subregions as described by Gruner [5].

In this paper, the accuracy of Gruner's analysis is examined, and found to be good, by comparing his results with the results of two disparate techniques for determining the cutoff frequencies of a rectangular stripline. Furthermore, a closed-form expression which can be used to determine the cutoff frequency of the first higher order mode is presented. This expression, based on a transverse resonance formulation, extends the range of Gruner's cutoff frequency analysis by including the region of strip width of $0.9 \leqslant W/C < 1.0$, where W and C are dimensions indicated in Fig. 1. Last, a novel experiment was performed to verify the analytical predictions of the cutoff frequency.

II. Method of Moments Solution

Following the generalized procedure outlined in another paper by the authors [6], an electric field integral equation (EFIE) was written and solved by a method of moments technique to determine the cutoff frequency of the higher order modes in a rectangular stripline. In this procedure, the solution begins with formulating an EFIE for the unknown current $\bar{J}$ on the thin inner conductor in terms of the vector potential $\bar{A}$ and the scalar potential ϕ as

$$\bar{E} = -j\omega\bar{A} - \nabla\phi \tag{1}$$

where

$$\bar{A}(x) = \mu\int\bar{J}(x')G(x;x')\,dx'$$

$$\phi(x) = \frac{j}{\omega\epsilon}\int\nabla\cdot\bar{J}(x')G(x;x')\,dx'. \tag{2}$$

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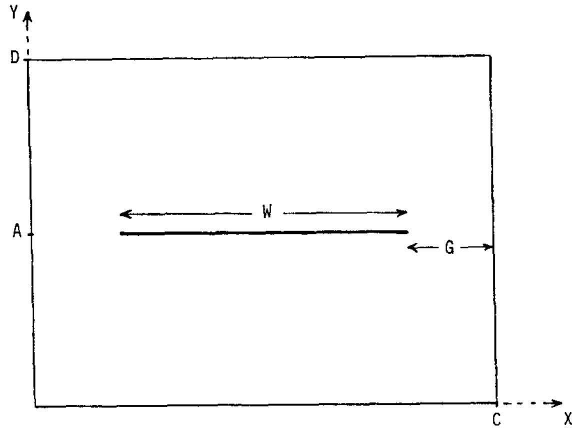

Fig. 1. Cross section of a rectangular stripline.

By applying the method of images, an infinite series, two-dimensional, free-space Green's function is written for this integral equation relating the unknown current on the inner conductor to the electric field inside the waveguide. Thus, the outer conducting wall of the rectangular stripline is replaced by a doubly infinite array of image sources, whose contribution to the field is incorporated in the Green's function.

This series Green's function, as is stands, converges very slowly. A numerically efficient solution to the EFIE requires the Green's function to be expressed in a more tractable form. As described in [6], the Poisson summation formula can be applied twice to convert the Green's function into a rapidly converging series in the Fourier transform domain. The resulting expression can be summed with respect to one index of summation in closed form. Kummer's transformation is then applied to accelerate the convergence of the remaining series. The Green's function, as developed above, is now expressed in a rapidly convergent form.

The use of triangle expansion functions to represent the unknown current, sampled with pulse testing functions, permits relatively simple expressions to be derived for filling the method of moments matrix [7]. By using this set of expansion and testing functions in conjunction with the described Green's function, this method of moments solution is demonstrated to be numerically efficient. Only 0.78 CPU seconds was required to fill and invert a (25×25) method of moments matrix on a Cyber 175.

III. Transverse Resonance Solution

In the case of the first higher order mode, a vertical E-wall can be placed bisecting the inner conductor in the rectangular stripline without disturbing the fields. With the presence of this E-wall, the rectangular stripline becomes a ridged waveguide with a ridge of infinitesimal thickness. The cutoff frequency of the first higher order mode of the ridged waveguide can be found by the transverse resonance technique described by Hopfer [8].

For the purpose of applying the transverse resonance technique, a capacitively loaded transmission-line models the ridged waveguide. The resonance condition for this model can be written as

$$2B = \operatorname{ctn}(k_0 A) + \operatorname{ctn}(k_0(D-A)) \tag{3}$$

where k_0 is the propagation constant of the medium, A and D are the dimensions indicated in Fig. 1, and B is the susceptance of a thin capacitive window in a parallel plate waveguide. A single-term representation for B determined by a variational

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technique [9] was found to give accurate results; that is,

$$B = \frac{2k_0 C}{\pi}\left\{\ln\left[\csc\left(\frac{\pi G}{C}\right)\right]\right\} \qquad (4)$$

where C and G are dimensions indicated in Fig. 1.

IV. Experiment

As in the case of the transverse resonance solution, an E-wall bisecting the rectangular stripline is inserted to create an equivalent guide for the first higher order mode. For a centered inner conductor, it is also recognized that an H-wall can be placed in the gap between the strip and the wall of the outer conductor. By inserting these E- and H-walls, a quarter-section of the rectangular stripline is isolated.

A novel experiment for accurately determining the cutoff frequency of the first higher order mode is derived from inspection of the isolated quarter-section of this rectangular stripline. At cutoff, the modal fields in this waveguide are z-independent and are transverse electric. It is observed that the complement of these fields corresponds to the transverse magnetic fields of a microstrip patch which is electrically thin in the z direction. In this complementary structure, the perfect electric conductor boundaries of the guide correspond to PMC boundaries of the microstrip patch, and the perfect magnetic conductor wall in the gap between the centered strip and the outer wall of the guide corresponds to an electrically shorted wall. Since they are complementary structures, the lowest resonant frequency of this microstrip patch corresponds exactly to the cutoff frequency of the first higher order mode of the rectangular stripline. On the basis of this correspondence, the cutoff frequency of the first higher order mode was measured by observing the resonances of microstrip patch antennas on a network analyzer.

The microstrip patch constructed for this experiment was 2.36-in square with a thickness of 0.132 in, centered on a large ground plane. The material used was 1-oz copper-clad Rexalite 220. The shorting of one side of the patch was accomplished with 22-gauge wire soldered at intervals of about 0.1 in. The resonant frequency of the patch was measured first without any pins and then, as pins were added, until an entire side of the patch was shorted. The effects of fringing were determined from comparing measurements of the resonant frequencies of the patch completely shorted along one side and completely without any shorting pins with the predictions of a simple cavity model of the patch. The patch antenna was excited with a coaxially fed probe. Since the patch antenna was thin, it had a very large Q and its resonance had a bandwidth of less than 1 percent, which resulted in an easily measured resonant frequency. The resonant frequency of the antenna was defined as the frequency where the reflection coefficient was at a minimum.

V. Results and Discussion

Table I compares the results of the two techniques and the experiment described in this paper with the results of Gruner's [5] analysis for the first cutoff frequency of a rectangular stripline. The agreement between all results is seen to be very close. Gruner's published results are verified, and the accuracy of the simple, closed-form transverse resonance equation is demonstrated. The experiment, besides its demonstrated accuracy, has the advantage of simplicity: the resonant frequency of a microstrip patch is easily determined with the aid of a network analyzer.

Field plots for several modes were generated using the method of moments technique. Inspection of these field plots led to the identification of the first higher-order mode as the TE_{11} odd coax mode, using the same nomenclature as is used in identifying elliptic coax modes [10]. The cutoff frequency of this coax mode degenerates to the rectangular waveguide TE_{10} mode as the inner conductor's width is reduced to zero. In none of the previous literature on rectangular striplines surveyed by the authors was a distinction made in modal nomenclature between the coax modes and rectangular waveguide modes. The authors believe that to uniquely and accurately define the modes, this distinction should be made. To be more specific, the waveguide modes have zero total current on the thin inner conductor, and the coax modes have current of the inner conductor. With rectangular stripline coax-type modes, as with elliptic coax modes, an odd or even designation should be made to indicate a mode's orientation in ϕ [10], [11].

TABLE I
COMPARISON OF CUTOFF FREQUENCIES FOR THE FIRST HIGHER ORDER MODE IN A RECTANGULAR STRIPLINE

W/C For C=D A=D/2	λ_c/C			
	Method of Moments	Transverse Resonance	Fourier Expansion [5]	Experiment
.20	2.065	2.07	2.062	2.06
.40	2.278	2.26	2.275	2.26
.60	2.665	2.64	2.666	2.63
.80	3.311	3.30	3.311	3.22

VI. Conclusion

Extensive graphs of cutoff frequencies for rectangular striplines are available in the literature [5], [6]. These results have been determined by the method of Fourier expansion of the fields in subregions. In this paper, these published results are verified by experimental evidence and by comparison with two alternative analytical techniques. A convenient closed-form equation defining a transverse resonance condition of rectangular striplines is presented. This expression allows the cutoff frequency of the first higher order mode to be easily determined, and it extends the range of available solutions for this problem. Last, a nomenclature scheme for uniquely identifying rectangular stripline modes is offered.

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